ACKNOWLEDGMENT

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NOTATION

A = amplitude of the rectangular wave perturbation input to the identification system
C_B = concentration of base in the stirred-tank, g-moles/l
c_B = C_B normalized with respect to C_{BM}, volume base/volume total solution
C_{BL} = concentration of base in the acidic (load) feed stream, g-moles/l
c_{BL} = C_{BL} normalized with respect to C_{BM}, volume base/volume feedstock
e = error
f = $\dfrac{G'}{(G+\overline{g})}$
g = flow rate of the control reagent, ml/min
G = flow rate of the acidic feedstock to the process tank, ml/min
G' = flow rate of the stream from process tank to the identification tank, ml/min
G_E = flow rate of the stream through the pH electrode chamber (Figure 3), ml/min
K = process gain
K' = gain of the identification system
$\hat{K}$ = estimate of K
$\hat{K}'$ = estimate of K'
K_C = controller gain
K_L = constant, defined by Equation (2c), ml feedstock/ml total solution
K_{LG} = loop gain (product of process gain and controller gain)
K_M = constant, defined by Equation (2b), (ml total solution/min)$^{-1}$
K_n = process gain during the n^{th} period of identification
K_n' = gain of the identification system during the n^{th} period of identification
$\hat{K}_n$ = estimate of K for the n^{th} period of identification
$\hat{K}_n'$ = estimate of K' for the n^{th} period of identification
S = slope of the neutralization curve at the operating point, V/(ml base/ml total solution)
S' = slope of the neutralization curve at the operating point, for the contents of the identification tank
t = time, s
T = process time constant, s
T' = time constant of the identification system, s
T_d = time delay in the measurement of the control variable, s
T_D = derivative time of the controller, s
T_I = integral time of the controller, s
T_n = period of identification for the n^{th} period, s
V = volume of solution in the process tank, ml
X = perturbation input to the identification system
x_p = manipulated variable
X_p = manipulated variable, deviation from the steady state
y = identification system output
Y = identification system output, deviation from the steady state
y_p = control variable
Y_p = control variable, deviation from the steady state
Y_{UL} = value of Y corresponding to Y_U or Y_L (Figure 4) for $Y_U = Y_L$
z_p = variable corresponding to c_B for the process

Superscripts

$-$ = bar over a variable indicates its steady state value
$*$ = deviation from the steady state

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Velocity Distributions in Die Swell

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Photographs of tracer particles were used to measure axial and transverse velocity components throughout the entire die swell region in a slit die. Laminar extrusion was investigated for two fluids: a viscoelastic concentrated solution of a polyacrylamide in glycerin and water and a highly viscous, nearly Newtonian, silicone oil. The die swell region extends upstream to the viscometric flow region and downstream to the relaxed portion of the extrudate. The data are used to evaluate several theories of die swell.

SCOPE

When viscoelastic fluids such as polymer melts and solutions are extruded through an orifice or from a die at low Reynolds number, the extrudate often reaches a cross-sectional area several times that of the die opening. For circular dies, the extrudate diameter can become two to four times larger than the die diameter (Bagley, 1963; Vlachopoulos, 1972). The phenomenon is usually called die swell. Die swell is an important parameter for design of shaping dies used in polymer processing operations such as extrusion, blow molding, and injection molding. For noncircular die geometries, the extrudate not only changes dimensions, but it changes shapes as well.

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Die designers would like to be able to design the size and shape of a die necessary to achieve a desired extrudate from first principles or engineering correlations along with a knowledge of the fluid properties. However, the complexity of viscoelastic fluid models and the difficulties posed by boundary value problems with unknown free boundaries combine to make predictions of die swell, even for the simplest geometries, a formidable problem.

Die swell theory can be broadly divided into four general categories: mass and momentum balances, elasticlike solid analyses, mathematical models (rheological models), and correlations with dimensionless groups. The analysis of Metzner et al. (1961), who used macroscopic mass and momentum balances to estimate normal stress differences, has been extended by other investigators using various assumptions about the velocity profiles at the exit. Some elastic analyses have been made based on the fact that die swell occurs very rapidly. The mathematical model approach requires rheological models to be solved simultaneously with the equations of motion, usually employing boundary-layer techniques.

A wealth of die swell data has been published, (Bagley, 1970; Vlachopoulus, 1972), principally for circular dies, and some progress has been made in interpreting swell of polymer melts at the molecular level, (Graessley et al., 1970; Vlachopoulus, 1972), but the literature provides no general correlations which the designer can use for arbitrary fluids or shapes.

In most analyses of die swell, assumptions must be made about the flow conditions at the die exit. Other approaches make assumptions about the velocity distribution throughout the entire die swell region. Unfortunately, experimental observations of these velocities are sparse.

In this investigation, time exposure photographs of tracer particles illuminated by flashing light were used to determine axial and transverse velocity components throughout the entire die swell region in a slit die for a viscoelastic and a nearly Newtonian fluid. The die swell region extends upstream to the viscometric flow region in the slit and downstream to the relaxed, free extrudate. Comparisons are made between the data and various theories.

CONCLUSIONS AND SIGNIFICANCE

At a Reynolds number and shear rate range found in many polymer processing operations, the viscoelastic fluid exhibits a definite exit effect. The fluid flow anticipates the die opening by developing a transverse velocity component from one-half to two-thirds of the die opening upstream. As the axial velocity component approaches the exit, the velocity profile becomes more blunt and, after passing the exit, becomes flat within a distance of one-fourth of the die opening. The die swell is not complete until about one die opening downstream. Inertial terms may be safely neglected in the macroscopic mass and momentum balances, which indicates that the average axial normal stress at the exit is equal to atmospheric pressure.

The principal significance of this work is the provision of the velocity distribution over the entire die swell region for well-characterized fluids, which will enable future investigators to evaluate proposed theories.

PREVIOUS WORK ON DIE SWELL: THEORY

Macroscopic Mass and Momentum Balances

Metzner et al. (1961) used macroscopic mass and momentum balances to infer normal forces from die swell measurements. By means of balances on the fluid between the capillary exit and a section downstream, where the velocities and stresses were constant across the section, a unique relationship between die swell and the first normal stress difference was established.

Their developed flow-at-exit assumption is not technically correct, but in Metzner's experiments the power law profile was quite flat in the upstream viscometric section and probably changed only slightly at the exit. Since the momentum balance analysis dealt with average velocities over a nearly flat profile, large errors would not be expected.

Graessley et al. (1970) applied Metzner's mass and momentum analysis to molten polystyrene and found that the normal stress differences determined by this method were several orders of magnitude smaller than those measured by a cone-and-plate rheometer.

Han and co-workers (1970*a*, 1970*b*, 1970*c*, 1971*a*, 1971*b*, 1971*c*) used the mass and momentum balance approach in conjunction with the exit pressure first noted by Arai (1968). The exit pressure was obtained by a linear extrapolation to the capillary exit of radial normal-force values measured along the capillary. There is considerable question about this method for obtaining normal forces (Tanner and Pipkin, 1969; Kearsley, 1970; Broadbent et al., 1968; Kay et al., 1968). Blum et al. (1971) also used the assumption of developed flow up to the exit in a similar analysis to determine normal stresses in a slit die. Bird (1974) has extended this analysis by including an energy balance which eliminates the necessity of evaluating the exit conditions. His work is discussed in a later section.

Most of the analyses of die swell include the assumption of developed flow up to the exit. Pressure measurements along a capillary by Balmer (1972) showed a very pronounced nonlinearity at the exit. Richardson (1970*a*, 1970*b*) predicted a definite exit effect in his mathematical development of die swell of Newtonian fluids. Vinogradov et al. (1972), Funatsu and Mori (1968), and Tordella (1969) all showed experimental evidence of shear stress changes and cessation of viscometric flow as the fluid approached the exit.

Hydrodynamic Approach

Another approach to the analysis of die swell consists of solving a constitutive equation simultaneously with the differential equations of conservation of mass and momentum. The complexity of reasonable constitutive equations and the difficulty of solving the resulting simultaneous integrodifferential equations have limited this approach to simple geometries, simple equations of state, or approximations in the solution.

One of the difficulties in analysis of die swell is the problem of applying the boundary conditions at the free surface, since this surface location is not known until the stress and velocity fields are determined. To facilitate the application of boundary conditions on the free surface, Duda and Vrentas (1967) introduced a protean coordinate system in which the streamlines were coordinate surfaces. They reported good agreement with experimental values for die swell of water jets at relatively high Reynolds numbers. Unfortunately, most polymer processing occurs at Reynolds numbers many orders of magnitude smaller.

Sagendorph and Leigh (1971) used the protean coordinate concept to predict exit flow from between parallel plates of a finite linear viscoelastic fluid model. Their results predicted a smaller than expected amount of die swell. Leigh (1973) explored pure swelling for the case where

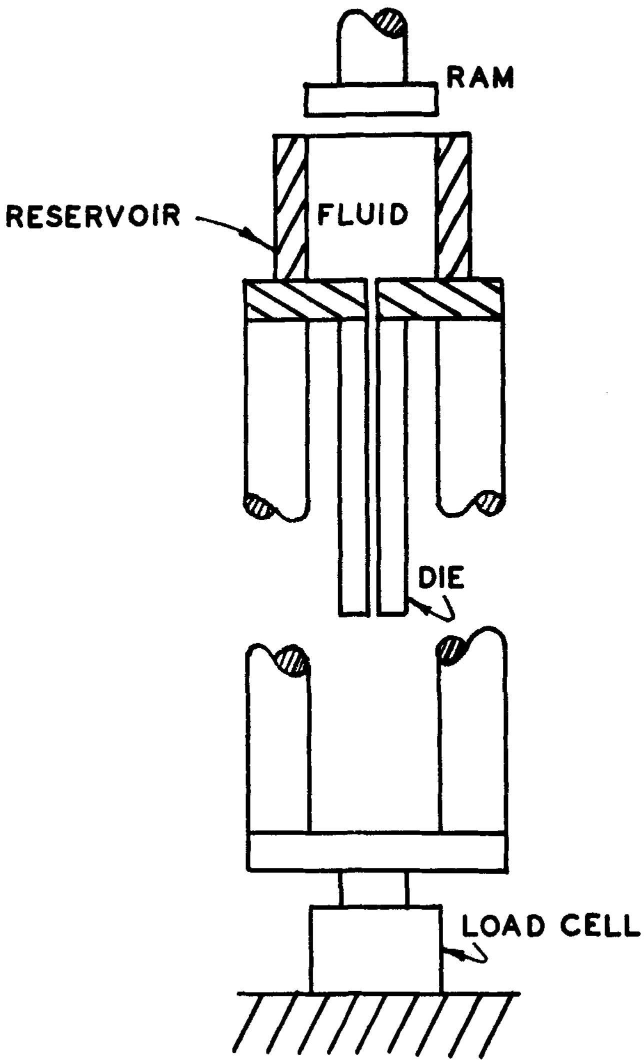

Fig. 1. Viscometer and transparent die.

the die walls were suddenly removed from a flowing finite linear viscoelastic fluid. His results suggested that the velocity profile will oscillate prior to stabilizing.

Horsfall (1973) used a finite-difference method of solving the equations of motion for an incompressible Newtonian fluid at zero Reynolds number subject to a zero normal stress at the extrudate surface. His predictions of the die swell ratio were approximately one half of those obtained experimentally.

Richardson (1970*a*, 1970*b*) examined Newtonian flow between flat plates in which the boundary conditions suddenly change from no slip to slip at the wall. This treatment simplifies the application of boundary conditions by fixing the free surface. Richardson found an analytical solution that indicated velocity disturbances (fluid particle accelerations) from approximately one die opening upstream to two openings downstream.

Elastic Recovery Concept

Since die swell generally occurs very rapidly, it suggests that an instantaneous elastic response may approximate the material's behavior. Nakajima and Shida (1966) analyzed die swell in terms of a rubberlike solid recovering from a tensile elongation after emergence from the die. Bagley and Duffey (1970) and Mendelson et al. (1971) proposed variations on Nakajima and Shida's analysis. MacIntosh (1961) calculated die swell as a function of recoverable shear from an average infinitesimal elastic shear strain, and Cogswell (1970) developed a relationship very similar to MacIntosh's with swell as a function of shear strain at the wall.

Consideration of swell as an elastic response has limitations. The stress on a viscoelastic fluid element is generally a function of the past rate of deformation. For instance, the decay of die swell with die length cannot be described by a strain history independent, elastic solid type of analysis. Lack of this past time strain dependence is a serious objection when a prediction of die swell is important. It is unlikely that the models on which recovery are based are capable of predicting die swell over anything but a limited range of operating conditions.

PREVIOUS MEASUREMENTS OF VELOCITY AND STRESS IN SWELLING JETS

Galt and Maxwell (1964) measured velocity profiles only in the developed flow region in circular glass capillaries by a tracer particle technique. Den Otter et al. (1967) performed experiments similar to those of Galt and Maxwell in rectangular slit geometry. Gogos and Maxwell (1966) measured velocity profiles of molten polyethylene near the exit of a circular glass capillary but with questionable accuracy (den Otter et al., 1967; Whipple, 1974). Rea and Schowalter (1967) measured velocity profiles between concentric cylinders, and Allen and Schowalter (1975) measured velocity profiles in polymer solutions in the exit region of a circular capillary.

To evaluate die swell analyses more completely, it would be desirable to measure complete stress distributions in the die swell region. There does not appear to be any reliable experimental method for doing so at present. Prados and Peebles (1959) used a birefringence technique to measure stress in a specially prepared Newtonian fluid in viscometric flow. Han and Drexler (1973) recently used the birefringence technique and particle tracer photographs to study the velocity and stress distributions in die entries.

Traditionally, circular dies have been used for die swell studies, but optical distortion through curved transparent dies is more severe than for plane-walled dies. Misalignment and focal plane depth contribute considerably more error in a circular section than a planar one. We have discussed the relative magnitude of these errors elsewhere (Whipple, 1974; Whipple and Hill, 1978).

EXPERIMENTAL PROGRAM

A transparent die with 50.8 by 3.2 mm slit and L/h of 80:1, constructed from transparent acrylic plastic, was attached beneath a large fluid reservoir. The test fluid containing tracer particles exited vertically from the die when driven by the downward movement of an Instron testing machine crosshead (see Figure 1).

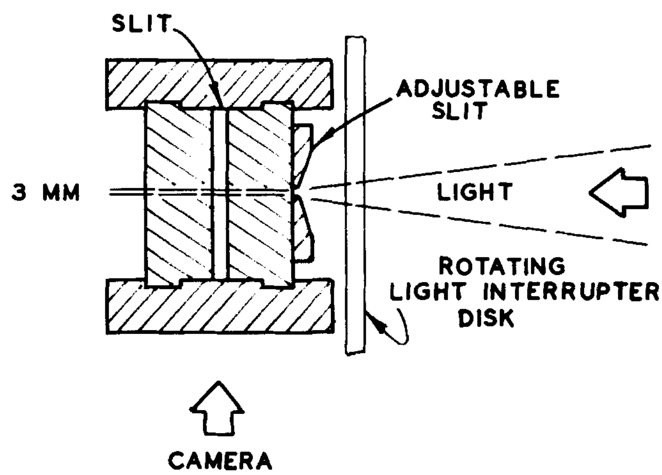

Fig. 2. Die cross section and light path.

The fluid and tracer particles were illuminated by a tungsten-halogen light source interrupted by a rotating disk, and time exposure photographs were taken of the tracer particle paths as shown in Figure 2. Exposure times were chosen to produce short streaks with minimum curvature.

To stabilize the extrudate and prevent the side to side oscillations that occur as the extrudate collects outside the die, a pair of driven rollers with adjustable gap, speed, and height were constructed and located as shown in Figure 3. To minimize the effects of gravity on the extrudate, the rollers were adjusted until the sides of the extrudate remained parallel after the initial swelling near the exit.

One of the problems of studying the velocity fields in the die swell region beyond the exit is the distortion caused by the curvature of the extrudate surface. In our work we extended the sides of the die wall at the edge of the slit so that the entire die swell region could be observed through planar interfaces. Extending the sides prohibited swell in the width direction but did not significantly affect the velocity field or swell in the thickness direction near the center of the slit (Whipple, 1974).

A camera, positioned so that the lens axis coincided with the center line of the die opening at the exit, was focused on a plane at the center of the slit section. Photos of a grid located in the flowing stream were made prior to each run for position calibration.

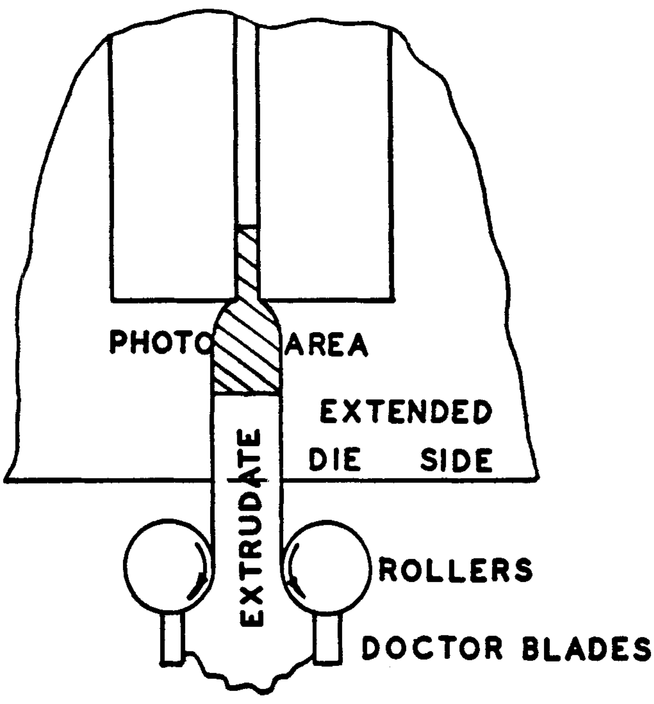

Fig. 3. Roller and die arrangement.

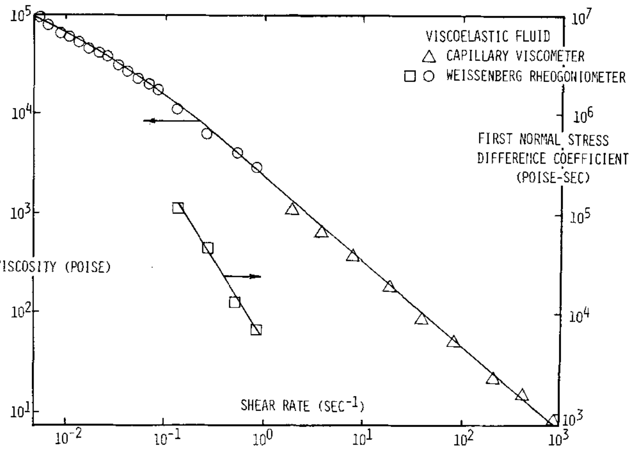

Fig. 4. Flow data for viscometric fluid.

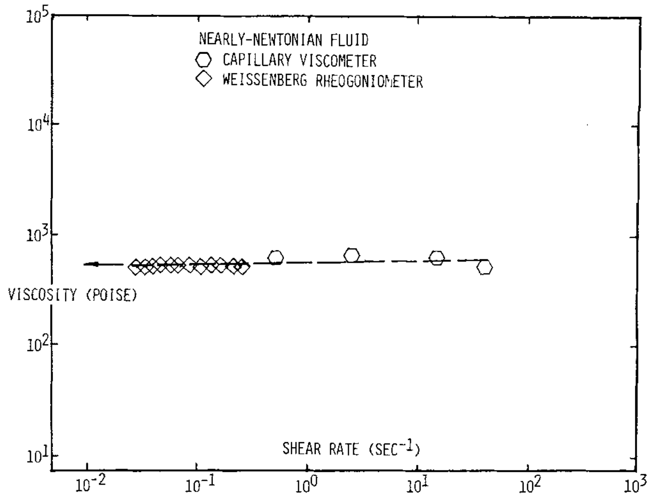

Fig. 5. Flow data for nearly Newtonian fluid.

Two fluids were used, a nearly Newtonian silicone oil (Dow Corning 200 Fluid) and a viscoelastic fluid composed of 47.6 wt % water, 47.6 wt % glycerine, and 4.7 wt % Separan AP 30 (Dow Chemical Co.). Rheological properties of the fluids measured on a Weissenberg rheogoniometer at the University of Houston are shown in Figures 4 and 5.

DATA ANALYSIS

Enlarged tracer streak photographs were used to locate the coordinates of the streaks and the die edges on an arbitrary X-Y coordinate system. A computer program converted the coordinates to a rectangular system with origin at the die exit center, $-Y$ in the axial flow direction, and X in the transverse direction. The coordinates, scaled to the

TABLE 1

Run number	1	2	3	4	5
Fluid type	VE	VE	VE	VE	NN
Instron ram speed	0.05 cm/min	0.05 cm/min	0.05 cm/min	0.50 cm/min	0.50 cm/min
Flow rate (cm^3/s)	0.164	0.162	0.170	1.67	1.77
Reynolds number	7×10^{-6}	7×10^{-6}	7×10^{-6}	8×10^{-5}	7×10^{-4}
Interrupter rev/min	96	95.5	105	970	822
Roller rev/min	1.28	0.88	0.32	18	23.5
Temperature	26°C	25°C	25°C	25°C	26°C
Die extension length	5.08 cm	3.81 cm	2.54 cm	2.54 cm	2.54 cm
Distance from die exit to rollers	6.35 cm	5.08 cm	3.81 cm	3.81 cm	3.81 cm

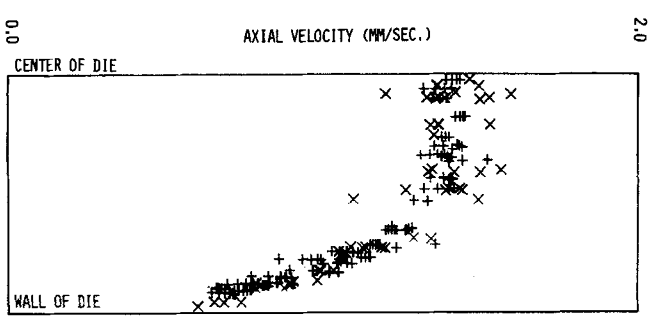

Fig. 6. Typical data.

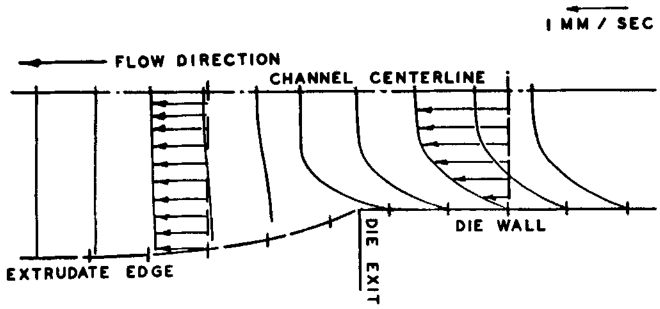

Fig. 8. Axial velocity component for nearly Newtonian fluid, run 5.

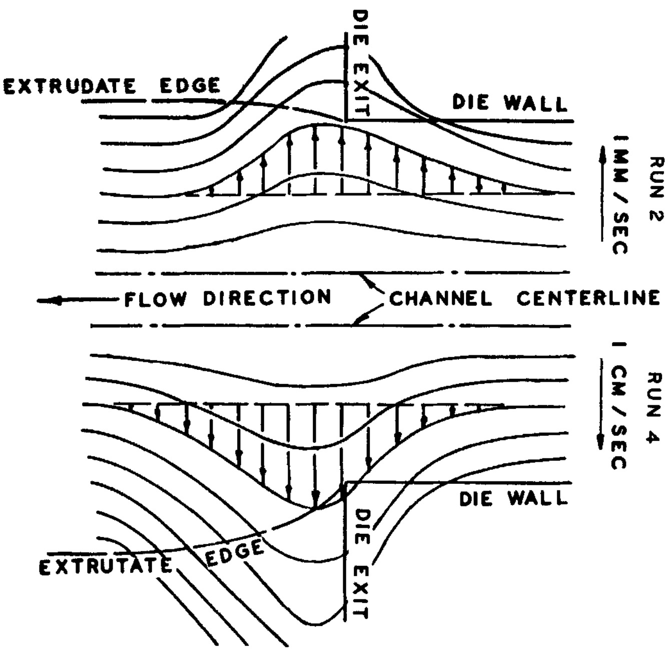

Fig. 9. Transverse velocity component for viscoelastic fluid, run 2 (slow) and run 4 (fast).

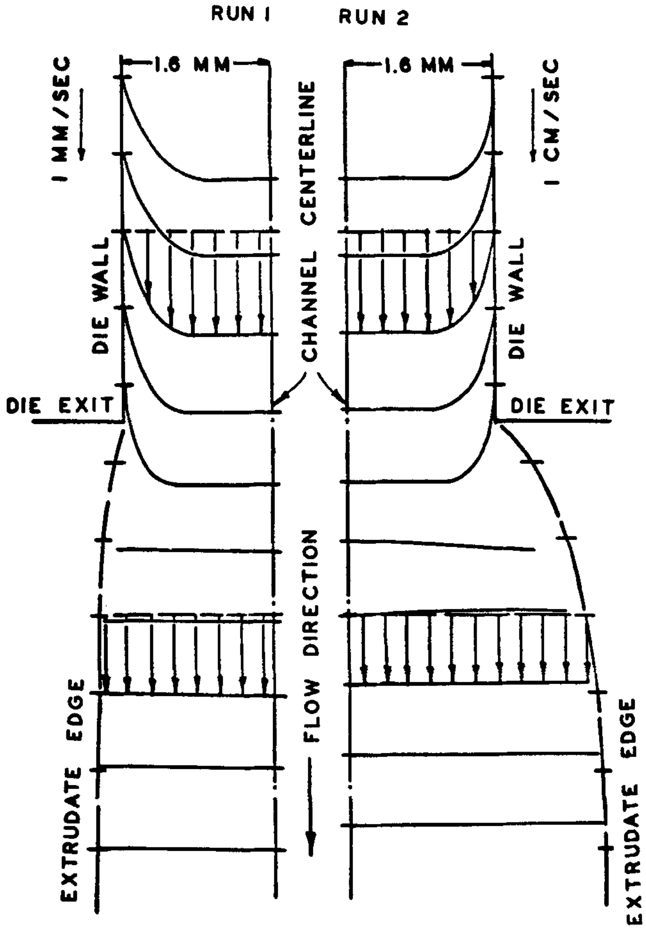

Fig 7. Axial velocity component for viscoelastic fluid, run 2 (slow) and run 4 (fast).

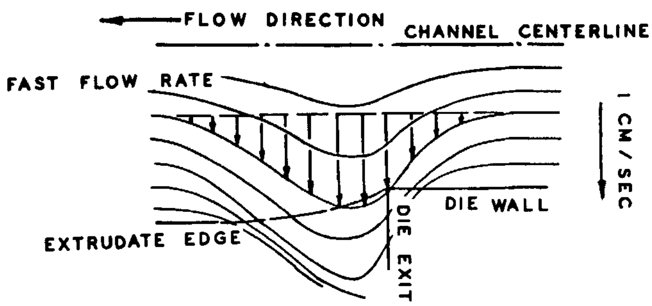

Fig. 10. Transverse velocity component for nearly Newtonian fluid, run 5.

die width and corrected for optical distortion, were then used to calculate axial and transverse velocity components. Finally, axial velocity values within sections of length equal to one-fourth the die width or 0.8 mm were assumed to represent the velocity at the axial position of the center of each section. Likewise, transverse velocities within sections of width 0.2 mm were assumed to represent the velocity at the transverse position of the center of each section.

The axial velocity components in each section were fit to a generalized power law profile of the form

$$v_y = A + B\,(1 - x^C) \tag{1}$$

with parameters A, B, and C dependent on y. In the die, $A = 0$, and B represents the center line velocity.

Attempts were made to fit the transverse velocities to polynomials of various order, but the scatter of the data precluded a statistically significant fit.

EXPERIMENTAL RESULTS

Table 1 summarizes the experimental conditions for five runs reported here. Reynolds numbers for the viscoelastic fluid were calculated based on the standard power law generalization as given in Bird et al. (1960).

Figure 6 is typical of the axial velocity data obtained, and Figures 7 to 10 are plots of the data smoothed by fitting to Equation (1) for runs 2, 4, and 5. Curve fit parameters for Equation (1) are shown in Tables 2 to 4, along with the standard deviations from the assumed functional form. These errors ranged from 3 to 7%, with most slightly above 5%. The maximum expected error in the

Table 2. Axial Velocity Curve Fit Parameters for Run 2

Section	A	B	C	% error
1	0.000	0.055	5.0	8.0
2	0.000	0.054	5.2	4.7
3	0.000	0.054	5.6	4.0
4	0.000	0.055	5.5	4.1
5	0.000	0.055	6.0	4.6
6	0.000	0.055	6.8	4.6
7	0.000	0.054	7.8	4.8
8	0.000	0.053	10.6	5.9
9	0.000	0.051	13.2	6.4
10	0.000	0.048	10.8	7.6
11	0.044	0.002	0.0	7.6
12	0.041	0.003	0.2	3.7
13	0.042	0.002	0.3	5.7
14	0.041	0.002	0.2	4.0
15	0.039	0.003	0.4	5.2
16	0.038	0.005	0.5	6.2
17	0.040	0.004	0.4	8.3
18	0.040	0.004	0.4	6.7

Each section following section 1 contains the points for a distance of one-fourth of the die opening.

Table 3. Axial Velocity Curve Fit Parameters for Run 4

Section	A	B	C	% error
1	0.00	0.53	11.8	6.3
2	0.00	0.53	7.1	6.8
3	0.00	0.53	6.7	6.2
4	0.00	0.52	7.0	6.5
5	0.00	0.50	9.9	7.6
6	0.44	0.02	−0.1	7.6
7	0.37	0.01	0.2	5.6
8	0.34	0.01	−0.1	5.1
9	0.30	0.01	−0.3	4.9
10	0.27	0.02	0.1	6.7
11	0.28	0.01	−0.1	6.3
12	0.29	0.01	−0.4	3.3

Each section following section 1 contains the points for a distance of one-fourth of the die opening.

Table 4. Axial Velocity Curve Fit Parameters for Run 5

Section	A	B	C	% error
1	0.00	0.64	3.2	4.0
2	0.00	0.64	2.2	4.7
3	0.00	0.63	3.2	5.0
4	0.00	0.63	3.9	5.6
5	0.00	0.61	5.0	6.0
6	0.40	0.10	9.7	6.0
7	0.40	0.05	8.0	6.3
8	0.40	0.02	6.3	3.6
9	0.39	0.01	0.0	2.8
10	0.39	0.01	0.1	2.6
11	0.40	0.01	0.1	2.7
12	0.40	0.01	0.1	2.2

Each section following section 1 contains the points for a distance of one-fourth of the die opening.

position of a coordinate point due to accumulated calibration error was estimated to be ± 3% of the true position. A complete set of graphs similar to Figure 6 are available in Whipple (1974).

Figures 7 to 10 indicate the existence of an exit effect, that is, of a flow disturbance upstream from the die exit. Although they were less accurate than the axial velocity curves, the transverse velocities were more sensitive to the exit effect. From Figures 9 and 10 the upstream disturbance distance was approximately two-thirds of the die opening for both flow rates of the viscoelastic fluid and about one-half of the die opening for the nearly Newtonian fluid.

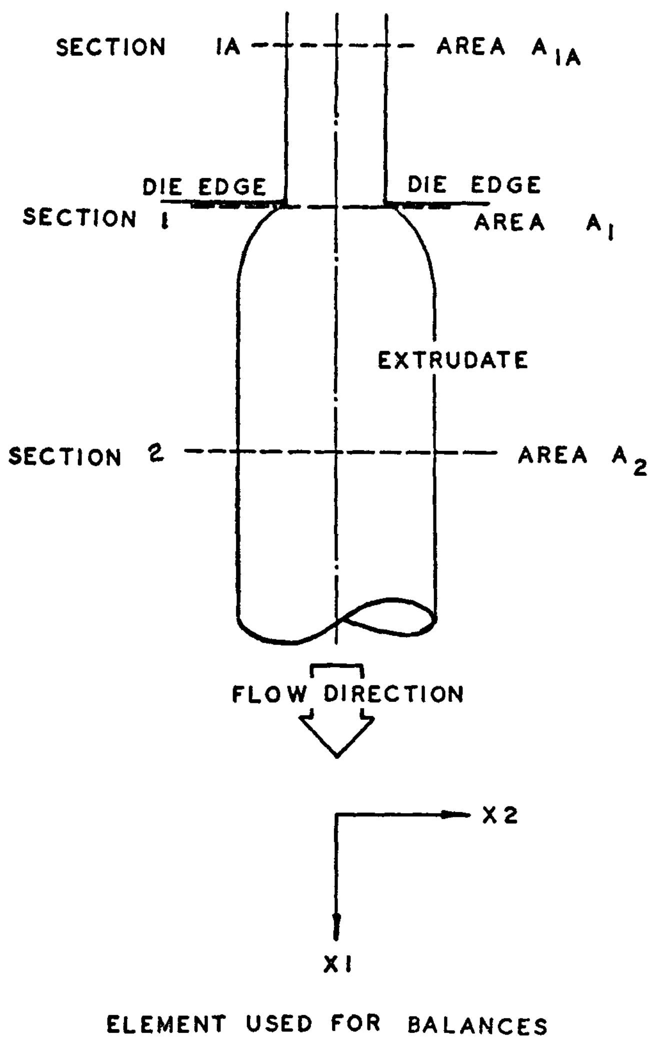

Fig. 11. Extrudate element with sections used for mass and momentum balances.

The axial velocity profile became more pluglike as it approached the exit. The slope of the axial velocity profile at the wall was also changing prior to the exit, especially for the viscoelastic fluid. A rapid rearrangement occurred just outside the die, and the axial velocity became constant across the extrudate prior to complete swelling. In Figure 7, for the fast flow rate, note the reverse curvature of the first profile outside the die and then the opposite curvature in the next profile indicating a velocity overshoot. This overshoot is also apparent in the raw data.

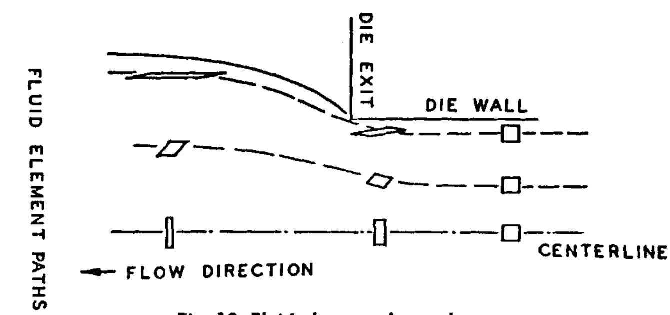

Fig. 12. Fluid element shape changes.

Figure 7 shows the axial velocity profiles for flow rates differing by an order of magnitude. The flow rate and the velocity scale of the right-hand side are ten times that of the left-hand side.

Except for the magnitude of the swelling, the nearly Newtonian fluid behaves similarly to the viscoelastic fluid. The magnitude of the transverse velocity near the outer edge of the extrudate exhibits the greatest difference between the two fluids.

The die swell ratio for the nearly Newtonian fluid of Figure 8 is approximately 1.7 for a Reynolds number of 7×10^{-4}. For comparison, Gavis and Modan (1967) found a swell ratio for Newtonian fluids exiting circular dies of 1.1 to 1.2 at Reynolds numbers around 1.0. Their data suggest that even higher ratios might have been observed at the very low Reynolds numbers we employed.

THEORETICAL IMPLICATIONS OF THE VELOCITY PROFILE DATA

Our experimental velocity profiles can be used to make preliminary evaluations of several approaches to prediction of die swell phenomena.

A mass balance and a momentum balance in the flow direction on the element of fluid between the die exit and a section of constant extrudate size [sections (1) and (2) in Figure 11] yield the following expression for the average axial normal stress at the die exit:

$$<\pi_{11}>_1 = \frac{\rho<v_1^2>_1}{B}(K - B) + p_a \qquad (2)$$

where

$$B = A_2/A_1 \quad \text{and} \quad K = <V_1>_1^2/<V_1^2>_1$$

Using the information from our experiments, the value of K is within 10% of unity, and the greatest value for $\rho<v_1^2>_1$ is about 1 dyne/cm^2. In our case, then the average axial normal stress at the exit is

$$<\pi_{11}>_1 \doteq p_a \qquad (3)$$

For molten polymers, the value of K will be nearly unity, since the velocity profiles are quite blunt. The value p_a is about 10^6 dynes/cm^2, which means that the velocity must approach 750 cm/s before it affects the value of $<\pi_{11}>_1$ by more than 10%. With the possible exceptions of high speed wire coating and thin film extrusion, most polymer processing in which die swell is a factor occurs well below this speed.

In a mass and momentum analysis of die swell with our test fluids and with most polymers in conventional processing, it is possible to neglect inertial terms with little, if any, loss in accuracy in the relationship between die swell and the normal stress at the exit.

The mass and momentum balance approach was extended by Bird (1974) to include a mechanical energy balance on the fluid element between sections (2) and (1A). His expression for die swell is

$$\frac{1}{B} = \alpha - \left\{ H - \theta + \frac{p_a(\alpha - 1)}{1/2\,\rho<v_1>_{1A}^2} - X \right\}^{1/2} \qquad (4)$$

in which

$$\alpha = \frac{<\pi_{22}\, v_1>_{1A}}{<\pi_{22}>_{1A}\,<v_1>_{1A}} \qquad (5)$$

H is a function of α and ratios of averages of the velocity at section (1A); θ is a function of α, velocity averages, and averages of the product of the first normal stress coefficient with shear rate at section (1A); and

$$-X = \frac{\alpha\, \bar{\tau}_w\, A_w}{A_{1A}(1/2)\,\rho<v_1>_{1A}^2} - e_{ve} \qquad (6)$$

where e_{ve} is a friction loss factor (Bird et al., 1960) which indicates the relative importance of the absorbed energy in the fluid element that is being dissipated as heat or contributing to the die swell. From an order of magnitude estimate based on our experimental results, B depends strongly on X which, in turn, is found as the difference of two large numbers. The first term for $-X$ changes by an order of magnitude, while die swell changes by less than 20%. Thus, the friction loss factor e_{ve} is a sensitive indicator of die swell in our case, and further work on correlations of e_{ve} with die swell might prove useful.

Many of the rubberlike elasticity models implicitly define the change in shape of the fluid elements as they pass from the capillary. A qualitative description of the shape change of a fluid element in our experiments as sketched in Figure 12 may aid in evaluating these models. An element, starting as a square at the center line of the channel, decelerates as it passes through the exit region. The once-square element ends up as a rectangle with its short side in the flow direction. A similar element starting at some position between the center line and the wall of the die will be sheared into a parallelogram by the velocity gradient while it is first accelerated and then decelerated to its final swelled shape. During the acceleration and deceleration, the element shifts away from the center line. The final shape is approximately a parallelogram with the short side aligned parallel to the flow direction. Near the wall, the same starting shape will be accelerated to its final swelled-extrudate speed while experiencing a shearing force. During acceleration, the element shifts away from the center line. This element near the wall will have an extended, nearly parallelogram final shape with the long side nearly parallel to the flow. These shape changes are grossly different from those which are assumed to occur in the elasticlike fluid theories, and our data thus throw considerable doubt on those approaches at a fundamental level.

Finally, these experimental data should aid in calculation of die swell from the differential equations of motion and realistic constitutive equations, because they provide detailed velocity field information from which to begin, or with which to test the results of such calculations.

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NOTATION

A, B, C = velocity curve fit parameters
A_{1A} = cross-sectional area at section 1A, Figure 8
A_W = interfacial area between sections 1A and 2, Figure 8
B = die swell, ratio of die and extrudate area or widths
K = ratio of average square to the square of the average of the velocity
p_a = atmospheric pressure
Re = Reynolds number, $vD\rho/\eta$
v_x = transverse velocity component
v_y = axial velocity component
$<v_1^2>_{1A}$ = average of the square of the velocity in the 1 direction at section 1A

$\langle v_1 \rangle^2_{1A}$ = average velocity in the 1 direction at section 1A squared
Vol = volume of fluid between sections 1A and 2
x = rectangular coordinate transverse to the flow direction
y = rectangular coordinate parallel to the flow direction

Greek Letters

$\underline{\underline{\dot{\gamma}}}$ = rate of deformation tensor, $\nabla v + \nabla v^{\dagger}$
$\underline{\nabla v}$ = gradient of the velocity
η = viscosity
π_{11} = normal stress in the 1 direction
π_{22} = normal stress in the 2 direction
$\langle \pi_{11} \rangle_{1A}$ = average normal stress in the 1 direction at section $1A$
ρ = fluid density
$\underline{\underline{\tau}}$ = deviatoric stress tensor
τ_w = average shear stress over area A_w

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